Brush Primitives Texture Transformations

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1 Notation

I'm using the subscript notation such that:

$$M = \begin{matrix} m_{11} & m_{12} & \cdots \\ m_{21} & m_{22} & \cdots \\ \vdots & \vdots & \ddots \end{matrix}$$

2 Overview

Firstly, I haven't checked that this algorithm works. It may be totally wrong. It may have subtle bugs. I disclaim all responsibility for anything that goes wrong with it, even if implementing it causes a rift in space-time through which dire beasts from the void come to ravage the Earth.

Let $\boldsymbol{n}$ be the normal vector of a polygon in the map's 3D space. Let d be the shortest distance from the polygon's plane to the origin of the map's 3D space.

Let $\boldsymbol{r}$ be a point in the map's 3D space lying within the polygon. Let $\boldsymbol{t}$ be the corresponding coordinate in 2D texture space. Then

$$\boldsymbol{t} = AB\boldsymbol{r}$$

Where B transforms $\boldsymbol{r}$ to a point in the 2D space defined by the polygon's plane, and A transforms that point into 2D texture space. Note that both A and B are square matrices of order 3.

Discarding $t_3$will render the UV coordinate corresponding to $\boldsymbol{r}$.

3 Origins

The origin of the map's 3D space is implicit.

The origin of the 2D space defined by the polygon's plane is the closest point on the plane to the map's origin.

The origin of the 2D space defined by the texture is congruent with the origin of the 2D space defined by the polygon's plane.

4 Transforming to polygon space

For any plane, there are an infinite number of ways to generate B, and all of them would give different results. The Brush Primitives format requires a particular algorithm[1]. Then

$$\theta = -\arctan\left(\frac{n_3}{\sqrt{n_1^2 + n_2^2}}\right)$$

$$\phi = \arctan\left(\frac{n_2}{n_1}\right)$$

$$B = \begin{bmatrix} -\sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & -\sin\theta\sin\phi & -\cos\theta \\ n_1 & n_2 & n_3 \end{bmatrix}$$

Note that the implementation must work when $n_1 = 0$.

5 Transforming to texture space

The matrix A is provided in the MAP file's brush face definition. After the three points defining the polygon's plane, the definition gives the first two rows of A, so that

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

This allows the MAP format to represent *any* linear transformation of the texture on the face.

[1] For optimisation, the bottom row of B could be $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. The composition as shown avoids throwing away any information.